

Determinations of quark mixing matrix elements $|V_{cd}|$ and $|V_{cs}|$ from leptonic and semileptonic D Decays

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Abstract

With the recent measurements of purely leptonic $D_{(s)}^+$ decays and semileptonic D decays in conjunction with decay constants $f_{D_{(s)}^+}$ and form factors $f_+^{\pi(K)}(0)$ calculated in LQCD, we extract the magnitudes of V_{cd} and V_{cs} to be $|V_{cd}| = 0.218 \pm 0.005$ and $|V_{cs}| = 0.987 \pm 0.016$. Compared to those given in PDG2013, the precisions of these newly extracted $|V_{cd}|$ and $|V_{cs}|$ are improved by more than 2.0 and 1.5 factors, respectively. With the newly extracted $|V_{cd}|$ and $|V_{cs}|$ together with other CKM matrix elements given in PDG2013, we check the unitarity of the CKM matrix, which are $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.997 \pm 0.002$, $|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.027 \pm 0.032$ and $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.023 \pm 0.032$.

1 Introduction

In the Standard Model (SM) of particle physics, the $D_{(s)}^+$ meson can decay into $\ell^+ \nu_\ell$ (where $\ell = e, \mu$, or τ) via annihilation mediated by a virtual W^+ boson. The decay rate depends upon the wave function overlap of the two quarks at the origin, which is parameterized by the $D_{(s)}^+$ decay constant, $f_{D_{(s)}^+}$. All of the strong interaction effects between the two initial-state quarks are absorbed into $f_{D_{(s)}^+}$. In the SM, the decay width of $D_{(s)}^+ \rightarrow \ell^+ \nu_\ell$ is given by

$$\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2 f_{D_{(s)}^+}^2}{8\pi} |V_{cd(s)}|^2 m_\ell^2 m_{D_{(s)}^+} \left(1 - \frac{m_\ell^2}{m_{D_{(s)}^+}^2}\right)^2, \quad (1)$$

where G_F is the Fermi coupling constant, $V_{cd(s)}$ is the $c \rightarrow d(s)$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element [1], m_ℓ is the lepton mass, and $m_{D_{(s)}^+}$ is the $D_{(s)}^+$ meson mass.

Similarly, in the SM, neglecting the positron mass, the differential decay rate of $D \rightarrow \pi(K)e^+\nu_e$ process is given by

$$\frac{d\Gamma}{dq^2} = X \frac{G_F^2}{24\pi^3} |V_{cd(s)}|^2 |\vec{p}_{\pi(K)}|^3 |f_+^{\pi(K)}(q^2)|^2, \quad (2)$$

where $\vec{p}_{\pi(K)}$ is the three-momentum of the π (K) meson in the rest frame of the D meson, $f_+^{\pi(K)}(q^2)$ represents the hadronic form factor of the hadronic weak current depending on square of the four-momenta transfer q^2 , and X is a factor due to isospin, which equals to 1 for $D^0 \rightarrow \pi^- e^+ \nu_e$, $D^0 \rightarrow K^- e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and equals to 1/2 for $D^+ \rightarrow \pi^0 e^+ \nu_e$. The form factor $f_+^{\pi(K)}(q^2)$ measures the probability to form the final state π (K) meson in this decay.

Recently, the branching fractions for leptonic D^+ and D_s^+ decays were well measured at the e^+e^- experiments near threshold of the $D\bar{D}$ production (CLEO-c and BESIII) and near 10.6 GeV (Belle and BaBar), and the decay constants f_{D^+} and $f_{D_s^+}$ were calculated in LQCD at precisions of $\sim 1.6\%$ and $\sim 1.1\%$, respectively. With these measured branching fractions in conjunction with the $f_{D_{(s)}^+}$ calculated in LQCD, the magnitudes of CKM quark mixing parameters V_{cd} and V_{cs} can be well extracted. In addition, the precisions of these measured branching fractions for $D \rightarrow \pi e^+ \nu_e$ and $D \rightarrow K e^+ \nu_e$ decays or measured products of $|V_{cd(s)}|$ and $f_+^{\pi(K)}(0)$ are at an accuracy level of about 1%, while the LQCD calculations of these form factors $f_+^\pi(0)$ and $f_+^K(0)$ also reach to about 4.4% and 2.5%, respectively. With these measured products of $|V_{cd(s)}|$ and $f_+^{\pi(K)}(0)$ together with inputs of the form factors calculated in LQCD, the magnitudes of V_{cd} and V_{cs} can also be well extracted.

In this article, we extract $|V_{cd}|$ and $|V_{cs}|$ with these measured branching fractions and/or $|V_{cd(s)}|f_+^{\pi(K)}(0)$ in conjunction with decay constants $f_{D_{(s)}^+}$ and/or form factors $f_+^{\pi(K)}(0)$ calculated in LQCD. In determinations of $|V_{cd}|$ and $|V_{cs}|$, we use G_F , masses of $D_{(s)}^+$ meson and leptons, and lifetimes of $D_{(s)}^+$ meson given in PDG2013 [1].

2 Recent experimental measurements

2.1 Purely leptonic D^+ decays

In 2008, the CLEO-c Collaboration accumulated 460055 ± 787 D^- tags by analyzing 818 pb^{-1} data taken at 3.773 GeV and selecting D^- mesons from 6 hadronic decay modes of the D^- meson. They observed 149.7 ± 12.0 signal events for $D^+ \rightarrow \mu^+ \nu_\mu$ decays in the system recoiling against these D^- tags. They measured the decay branching fraction $B(D^+ \rightarrow \mu^+ \nu_\mu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$ [2].

In 2014, the BESIII Collaboration measured the branching fraction for $D^+ \rightarrow \mu^+ \nu_\mu$ decays by analyzing 2.92 fb^{-1} data taken at 3.773 GeV. From 9 hadronic decay modes of

D^- meson, the BESIII Collaboration accumulated 1703054 ± 3405 D^- tags. In this D^- tag sample they observed 409.0 ± 21.2 signal events for $D^+ \rightarrow \mu^+ \nu_\mu$ decays and measured branching fraction $B(D^+ \rightarrow \mu^+ \nu_\mu) = (3.71 \pm 0.19 \pm 0.06) \times 10^{-4}$ [3].

Averaging these two branching fractions, we obtain

$$B(D^+ \rightarrow \mu^+ \nu_\mu) = (3.74 \pm 0.17) \times 10^{-4}, \quad (3)$$

where the error is the combined statistical and systematic errors together.

2.2 Purely leptonic D_s^+ decays

In 2009, the CLEO-c Collaboration studied the $D_s^+ \rightarrow \ell^+ \nu_\ell$ decays based on 600 pb^{-1} data taken at 4.17 GeV. From this data sample, they tagged D_s^- mesons from 9 hadronic decay modes. By examining distribution of missing mass-squared of the D_s^- and γ system they accumulated 43859 ± 936 D_s^+ mesons; by analyzing distribution of missing mass-squared of the $D_s^- \gamma \mu^+$ system, they selected $D_s^+ \rightarrow \mu^+ \nu_\mu$ decay events and measured the branching fraction $B(D_s^+ \rightarrow \mu^+ \nu_\mu) = (0.565 \pm 0.045 \pm 0.017)\%$ [4]. Using similar method, the CLEO-c Collaboration also measured the branching fraction $B(D_s^+ \rightarrow \tau^+ \nu_\tau) = (5.58 \pm 0.33 \pm 0.13)\%$, which is the average of three measured branching fractions obtained with $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$ [4], $\tau^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau$ [5] and $\tau^+ \rightarrow \rho^+ \bar{\nu}_\tau$ decays [6].

In 2013, the Belle Collaboration measured the branching fractions for leptonic D_s^+ decays. They selected leptonic D_s^+ decays from the $e^+ e^- \rightarrow c \bar{c}$ continuum production, in which the $D_{\text{tag}} K_{\text{frag}} X_{\text{frag}} D_s^{*+}$ is produced from the quark fragmentation, where $D_s^{*+} \rightarrow \gamma D_s^+$, K_{frag} is either K^+ or K_S^0 , and X_{frag} indicates several pions or photons. By reconstructing the recoil mass of the $D_{\text{tag}} K_{\text{frag}} X_{\text{frag}} \gamma$, they observed clear D_s^+ signal in the system recoiling against the $D_{\text{tag}} K_{\text{frag}} X_{\text{frag}} \gamma$. By fitting the recoil mass spectra of $D_{\text{tag}} K_{\text{frag}} X_{\text{frag}} \gamma$, they accumulated $94360 \pm 1310 \pm 1450$ inclusive D_s^+ mesons. To search for $D_s^+ \rightarrow \mu^+ \nu_\mu$ decays, they examined the missing mass-squared $M_{\text{miss}}^2(D_{\text{tag}} K_{\text{frag}} X_{\text{frag}} \gamma \mu)$ distribution of the $D_{\text{tag}} K_{\text{frag}} X_{\text{frag}} \gamma \mu$ system. Fitting the $M_{\text{miss}}^2(D_{\text{tag}} K_{\text{frag}} X_{\text{frag}} \gamma \mu)$ distribution yields 492 ± 26 signal events for $D_s^+ \rightarrow \mu^+ \nu_\mu$ decays. With these numbers of events, the Belle Collaboration measured the decay branching fraction $B(D_s^+ \rightarrow \mu^+ \nu) = (0.531 \pm 0.028 \pm 0.020)\%$ [7]. In addition, the Belle Collaboration observed 2217 ± 83 signal events for $D_s^+ \rightarrow \tau^+ \nu_\tau$ decays with $\tau^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau$, $\tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau$ and $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$ decays, and measured the decay branching fraction $B(D_s^+ \rightarrow \tau^+ \nu_\tau) = (5.70 \pm 0.21_{-0.30}^{+0.31})\%$ [7].

In 2010, using the similar technique as the one used by the Belle Collaboration, the BaBar Collaboration made measurements of the branching fractions for leptonic D_s^+ decays. By analyzing 521 fb^{-1} data taken at 10.6 GeV, the BaBar Collaboration measured the decay branching fractions $B(D_s^+ \rightarrow \mu^+ \nu_\mu) = (0.602 \pm 0.038 \pm 0.034)\%$ and $B(D_s^+ \rightarrow \tau^+ \nu_\tau) = (5.00 \pm 0.35 \pm 0.49)\%$ [8].

Combining these branching fractions measured by the CLEO-c, Belle and BaBar Collaborations, we obtain

$$B(D_s^+ \rightarrow \mu^+ \nu_\mu) = (0.556 \pm 0.025)\% \quad (4)$$

and

$$B(D_s^+ \rightarrow \tau^+ \nu_\tau) = (5.54 \pm 0.24)\%, \quad (5)$$

where the errors are the combined statistical and systematic errors together.

2.3 Semileptonic D decays

In 2008, the CLEO-c Collaboration studied the semileptonic decays of $D^0 \rightarrow \pi^- e^+ \nu_e$, $D^0 \rightarrow K^- e^+ \nu_e$, $D^+ \rightarrow \pi^0 e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ by analyzing 818 pb⁻¹ data taken at 3.773 GeV. They extracted the products $f_+^\pi(0)|V_{cd}| = 0.150 \pm 0.004 \pm 0.001$ and $f_+^K(0)|V_{cs}| = 0.719 \pm 0.006 \pm 0.005$ by fitting their measured partial decay rates with form factor parameterized with three parameter series expansion [9].

Recently, the BESIII Collaboration reported their new preliminary results of $D^0 \rightarrow \pi^- e^+ \nu_e$ and $D^0 \rightarrow K^- e^+ \nu_e$ decays obtained by analyzing 2.92 fb⁻¹ data taken at 3.773 GeV. They obtained $f_+^\pi(0)|V_{cd}| = 0.1420 \pm 0.0024 \pm 0.0010$ and $f_+^K(0)|V_{cs}| = 0.7196 \pm 0.0035 \pm 0.0041$ by fitting differential decay rates with the three parameter series expansion [10].

In 2007, the BaBar Collaboration measured the form factors $f_+^K(q^2)$ by analyzing 75 fb⁻¹ data collected at 10.6 GeV and determined $f_+^K(0) = 0.727 \pm 0.007 \pm 0.005 \pm 0.007$ [11]. Multiplying this form factor by $|V_{cs}| = 0.9729 \pm 0.0003$ used in their paper, we obtain the product $f_+^K(0)|V_{cs}| = 0.707 \pm 0.007 \pm 0.005 \pm 0.007$. Using the same technique, the BaBar Collaboration also studied the $D^0 \rightarrow \pi^- e^+ \nu_e$ decay by analyzing 347.2 fb⁻¹ data collected at $\Upsilon(4S)$ and reported preliminary results at ICHEP2014. They measured $f_+^\pi(0)|V_{cd}| = 0.1374 \pm 0.0038 \pm 0.0022 \pm 0.0009$ [12].

Combining these $f_+^{\pi(K)}(0)|V_{cd(s)}|$ measured at the CLEO-c, BESIII and BaBar experiments, we obtain

$$f_+^\pi(0)|V_{cd}| = 0.143 \pm 0.002 \quad (6)$$

and

$$f_+^K(0)|V_{cs}| = 0.718 \pm 0.004, \quad (7)$$

where the errors are the combined statistical and systematic errors together.

3 Determinations of $|V_{cd}|$

Before 2012, the CKM matrix element $|V_{cd}|$ was usually determined with the $\nu\bar{\nu}$ interaction or the semileptonic decay of $D \rightarrow \pi e^+ \nu_e$. Actually, using the measured branching fraction for $D^+ \rightarrow \mu^+ \nu_\mu$ decays in conjunction with the LQCD calculation on D^+

meson decay constant, the magnitude of V_{cd} can also be extracted via the Eq. (1). At Charm2012, the BESIII Collaboration reported preliminary result on the determination of $|V_{cd}|$ based on their measured branching fraction for $D^+ \rightarrow \mu^+ \nu_\mu$ decay, which is $|V_{cd}| = 0.2218 \pm 0.0062 \pm 0.0047$ [13]. Recently, the Flavor Lattice Averaging Group (FLAG) made an average of several values of the f_{D^+} calculated in LQCD. The averaged D^+ decay constant calculated in LQCD is $f_{D^+} = (209.2 \pm 3.3)$ MeV [14]. Inserting the averaged branching fraction for $D^+ \rightarrow \mu^+ \nu_\mu$ decays as given in Eq. (3) and this averaged f_{D^+} into Eq. (1) yields

$$|V_{cd}|_{D^+ \rightarrow \mu^+ \nu_\mu} = 0.219 \pm 0.005 \pm 0.004, \quad (8)$$

where the first uncertainty is from the measured branching fractions and the second mainly from the uncertainties of f_{D^+} and the lifetime of D^+ meson.

Dividing the averaged $f_+^\pi(0)|V_{cd}|$ from semileptonic $D \rightarrow \pi e^+ \nu_e$ decays by the form factor $f_+^\pi(0) = 0.666 \pm 0.029$ calculated in LQCD [15] yields

$$|V_{cd}|_{D \rightarrow \pi e^+ \nu_e} = 0.215 \pm 0.003 \pm 0.009, \quad (9)$$

where the first uncertainty is from the measured $f_+^\pi(0)|V_{cd}|$, and the second uncertainty is from $f_+^\pi(0)$.

Figure 1 shows the comparison of $|V_{cd}|$ determined from purely leptonic D^+ decay and semileptonic D decay. Averaging the determined $|V_{cd}|_{D^+ \rightarrow \mu^+ \nu_\mu}$ and $|V_{cd}|_{D \rightarrow \pi e^+ \nu_e}$ yields

$$|V_{cd}| = 0.218 \pm 0.005. \quad (10)$$

Figure 2 shows the comparison of the newly determined $|V_{cd}|$ and the one given in PDG2013 [1].

4 Determinations of $|V_{cs}|$

Using the measured decay branching fractions for $D_s^+ \rightarrow \ell^+ \nu_\ell$ together with the D_s^+ meson decay constant calculated in LQCD, the magnitude of V_{cs} can be extracted via Eq. (1). We herein use the value of $f_{D_s^+} = (248.6 \pm 2.7)$ MeV, which is the FLAG average of several decay constants calculated in LQCD [14], to extract $|V_{cs}|$. Inserting the averaged branching fractions for $D_s^+ \rightarrow \ell^+ \nu_\ell$ decays and the $f_{D_s^+}$ into Eq. (1) yields

$$|V_{cs}|_{D_s^+ \rightarrow \mu^+ \nu_\mu} = 1.001 \pm 0.022 \pm 0.013 \quad (11)$$

and

$$|V_{cs}|_{D_s^+ \rightarrow \tau^+ \nu_\tau} = 1.011 \pm 0.022 \pm 0.013, \quad (12)$$

where the first uncertainties are from the measured branching fractions, and the second uncertainties are mainly from $f_{D_s^+}$ and the lifetime of D_s^+ meson. Combining the above two values, we obtain

$$|V_{cs}|_{D_s^+ \rightarrow \ell^+ \nu_\ell} = 1.006 \pm 0.016 \pm 0.013, \quad (13)$$

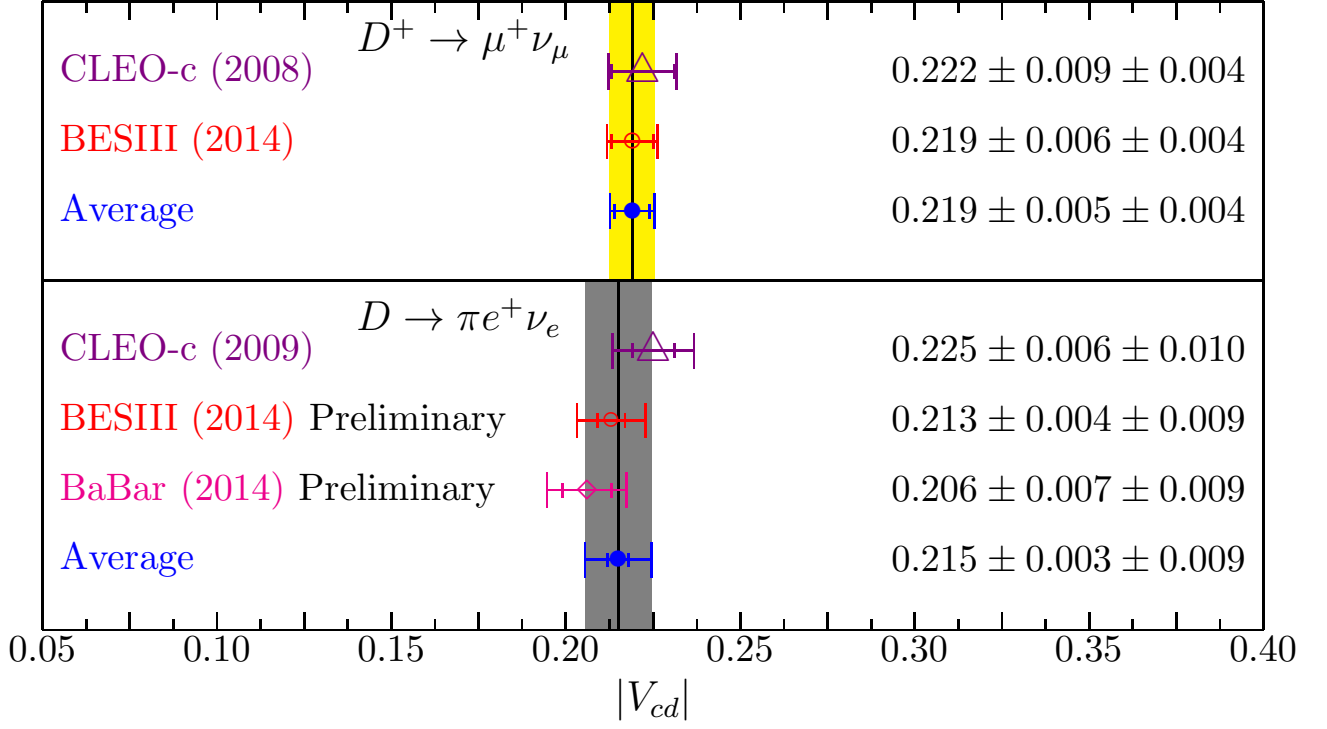


Figure 1: Comparison of $|V_{cd}|$ determined from leptonic D^+ and semileptonic D decays.

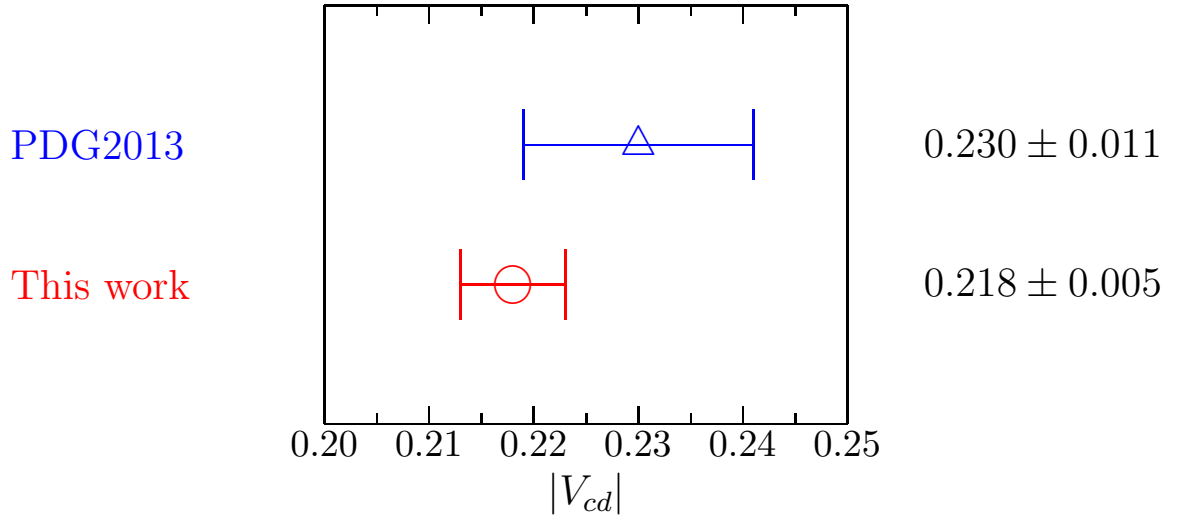


Figure 2: Comparison of the newly determined $|V_{cd}|$ from both the leptonic D^+ and semileptonic D decays and the one given in PDG2013.

where the first uncertainty is from the measured branching fractions, the second uncertainty is mainly from $f_{D_s^+}$ and the lifetime of D_s^+ meson.

Dividing the averaged $f_+^K(0)|V_{cs}|$ from semileptonic $D \rightarrow Ke^+\nu_e$ decays by the form factor $f_+^K(0) = 0.747 \pm 0.019$ calculated in LQCD [16] yields

$$|V_{cs}|_{D \rightarrow Ke^+\nu_e} = 0.961 \pm 0.005 \pm 0.024, \quad (14)$$

where the first uncertainty is from the measured $f_+^K(0)|V_{cs}|$ and the second uncertainty is from $f_+^K(0)$.

Figure 3 shows the comparison of $|V_{cs}|$ determined from purely leptonic D_s^+ decays and semileptonic D decays. Averaging the determined $|V_{cs}|_{D_s^+ \rightarrow \ell^+\nu_\ell}$ and $|V_{cs}|_{D \rightarrow Ke^+\nu_e}$ yields

$$|V_{cs}| = 0.987 \pm 0.016. \quad (15)$$

Figure 4 shows the comparison of the newly determined $|V_{cs}|$ and the one given in PDG2013 [1].

5 Unitarity checks

Using the newly extracted $|V_{cd}| = 0.218 \pm 0.005$, the PDG values $|V_{ud}| = 0.97425 \pm 0.00022$ and $|V_{td}| = (8.4 \pm 0.6) \times 10^{-3}$ [1], the first column unitarity of CKM matrix is checked, which is

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.997 \pm 0.002. \quad (16)$$

Using the newly extracted $|V_{cs}| = 0.987 \pm 0.016$, the PDG values $|V_{us}| = 0.2252 \pm 0.0009$ and $|V_{ts}| = (42.9 \pm 2.6) \times 10^{-3}$ [1], we find

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.027 \pm 0.032 \quad (17)$$

for the second column of the CKM matrix. Using these newly extracted $|V_{cd}|$ and $|V_{cs}|$, and the PDG value $|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$ [1], we find

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.023 \pm 0.032 \quad (18)$$

for the second row of the CKM matrix. The unitarity check results for the first column, second column and second row of the CKM matrix are shown in Fig. 5 together with the unitarity checks given in PDG2013 [1]. The newly determined $|V_{cd}|$ and $|V_{cs}|$ give more stringent checks of the CKM matrix unitarity compared to those in PDG2013.

6 Summary

Combining the precise measurements of leptonic $D_{(s)}^+ \rightarrow \mu^+\nu_\mu$ decays and semileptonic $D \rightarrow \pi(K)e^+\nu_e$ decays at the CLEO-c, Belle, BaBar and BESIII together with the improved

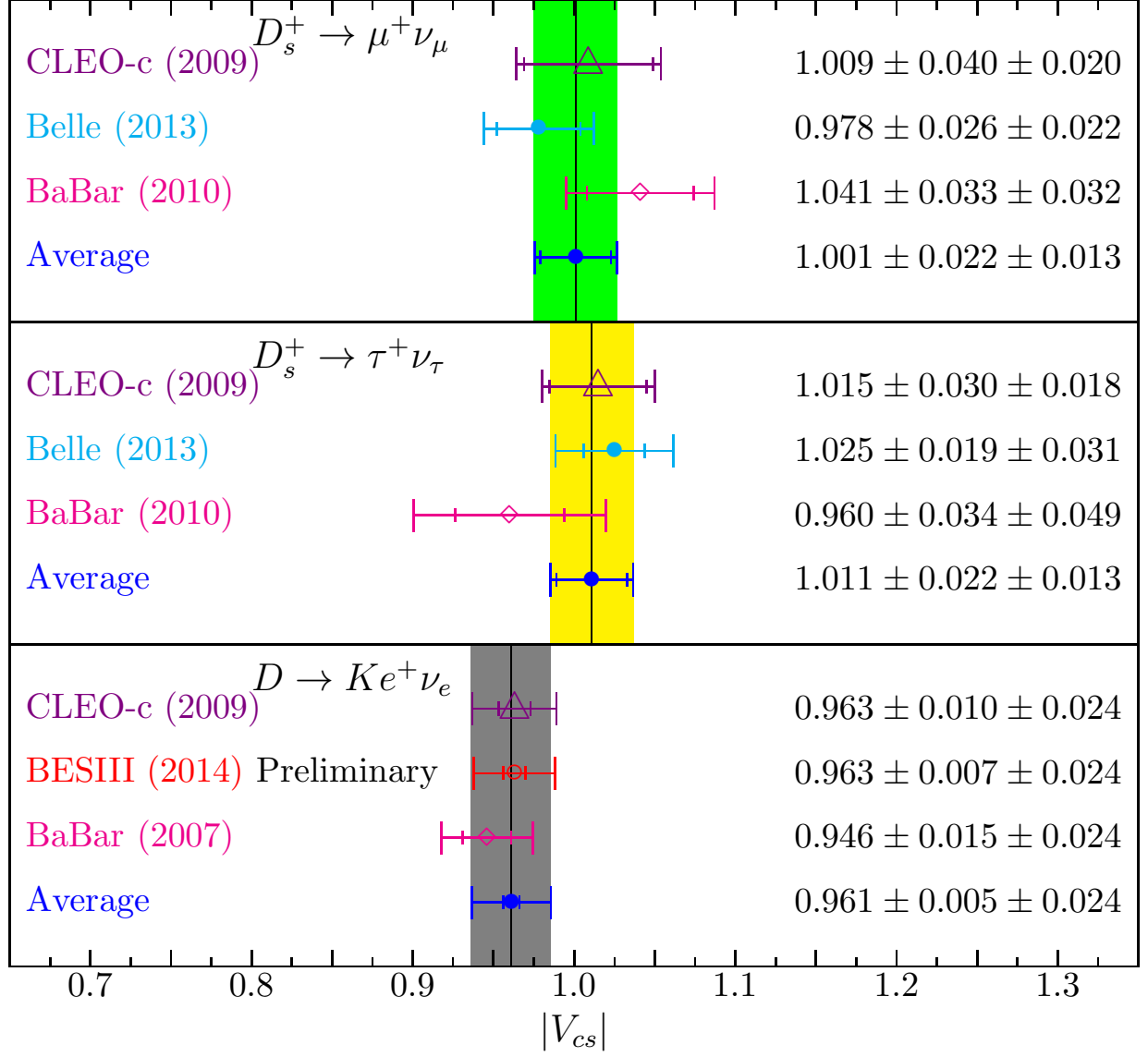


Figure 3: Comparison of $|V_{cs}|$ determined from leptonic D_s^+ decays and semileptonic $D \rightarrow Ke^+ \nu_e$ decays.

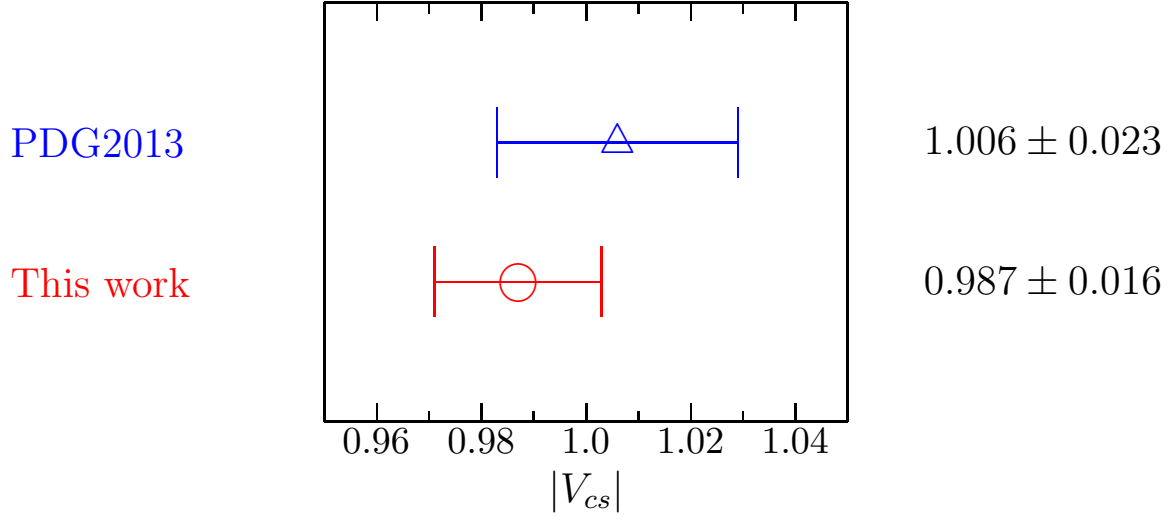


Figure 4: Comparison of the newly determined $|V_{cs}|$ from both the leptonic D_s^+ and semileptonic D decays and the one given in PDG2013.

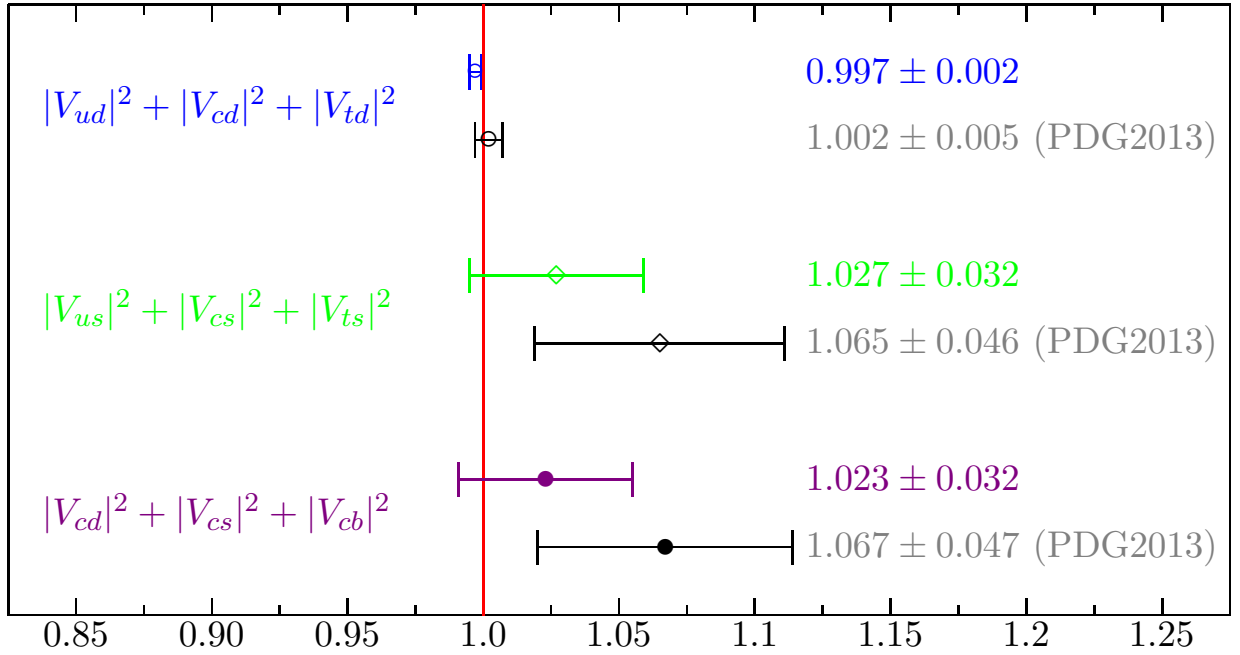


Figure 5: Unitarity checks for the first column, second column and second row of the CKM matrix.

$D_{(s)}^+$ decay constants and semileptonic D decay form factors calculated in LQCD, we extract the magnitudes of V_{cd} and V_{cs} to be $|V_{cd}| = 0.218 \pm 0.005$ and $|V_{cs}| = 0.987 \pm 0.016$, which improve the precisions of those values given in PDG2013 by more than 2.0 and 1.5 factors, respectively. These improved determinations of $|V_{cd}|$ and $|V_{cs}|$ give more stringent unitarity checks of the CKM matrix compared to those given in PDG2013.

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